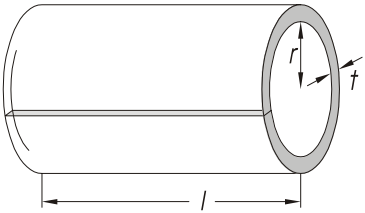
## Question

The maximum possible pressure in a tube is a function of material, diameter, and wall thickness. How does the necessary wall thickness change, if the diameter is varied?

## Answer $t = r \cdot p/s$

The necessary wall thickness $t$ is proportional to the radius $r$, to the pressure $p$ and inverse proportional to the tensile strength $s$.

**Proof** In advance: The tensile strength $s$ is the maximum possible strength per unit cross section of a material, before it is torn apart. Its unit is $N/m^2$.




Approach: Let us regard a slitted tube with length $l$, as shown on the left picture. Given the radius $r$ is increased by an infinitesimal d$r$ we may state the energy balance. The force perpendicular to the slit area ($l \cdot t \cdot s$), that is necessary to stretch the tube times stretching distance $2\pi \cdot dr$ equals exactly the energy that is added by the additional volume of the tube $(r+dr)^2 \cdot \pi \cdot l - r^2 \cdot \pi \cdot l$ at constant pressure $p$.

stretching work = energy added by additional volume
force $\cdot$ distance = volume $\cdot$ pressure

$$l \cdot t \cdot s \cdot 2\pi \cdot dr = [(r+dr)^2 - r^2] \cdot \pi \cdot l \cdot p$$

$$l \cdot t \cdot s \cdot 2\pi \cdot dr = 2r \cdot dr \cdot \pi \cdot l \cdot p \qquad (dr^2 \text{ is negligible small})$$

$$t \cdot s = r \cdot p$$

$$t = r \cdot p/s$$

**Example 1** Dealing with very high pressures it is good practice to use tubes with small diameters.

**Example 2** The necessary wall thickness decreases proportionally with the diameter and the pressure. For example blood capillaries with some micrometers in diameter and pressures in the range of 1000 Pa allow extreme thin walls.

**Example 3** If the pressure in the tube is comparable to the tensile strength ($p$ = s), the wall thickness equals about the radius of the tube ($t = r$). This is the case with fire arms. The gas pressure of rifle cartridges is between 1000 and 4000 bar, the pressure in canons reaches up to 7000 bar. The tensile limit of tempered steel of fire arms, however, ranges from 7500 to 12000 bar.