## Question

A swimmer is located at the center of a circular lake. A pursuer waiting on the banks of the lake can run four times faster than the swimmer in the water. On the land the swimmer could run faster than the pursuer. Is the swimmer able to escape?

Answer

She is. She manages to escape in two steps:

1. Starting at the center she swims on a spiral shaped path until she reaches the opposite side of the pursuer (look at the figure on the right side). The maximum extent of the spiral is $r_s = r_p/4$, because the **p**ursuer is four times faster than the **s**wimmer ($v_s = v_p/4$).
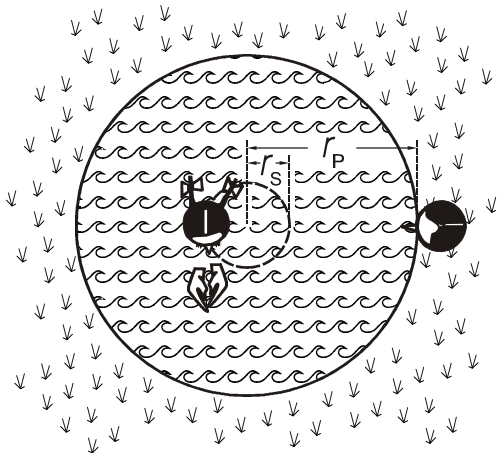
2. Having reached the position indicated in the figure she swims the shortest path possible to the left and runs away.
Assertion: She manages to do it, if she can swim the remaining distance in a shorter time t_s than it takes for her pursuer to run along the banks around the half lake (t_p), i.e., if $t_s < t_p$.
These times can be expressed by distances and velocities $(r_v - r_s)/v_s < (r_v \cdot \pi)/v_v$.
Using the equations from part 1. we have
$(4r_s - r_s)/v_s < (4r_s \cdot \pi)/4v_s$, and equivalently we get
$3 < \pi$.
Hereby the assertion is proved!